Marcin Adam
School of Labour Safety Management, Katowice, Poland

On the double quadratic difference property

Let X be a real normed space and Y a real Banach space. Denote by $C^n(X,Y)$ the class of n-times continuously differentiable functions $f\colon X \to Y$. We prove that the class C^n has the double quadratic difference property, that is if $Qf(x,y) := f(x+y)+f(x-y)-2f(x)-2f(y) \in C^n(X \times X, Y)$, then there exists exactly one quadratic function $K\colon X \to Y$ such that $f - K \in C^n(X,Y)$.

Mirosław Adamek
University of Bielsko-Biała, Poland

On two variable functional inequality and related functional equation

We present the result stating that the lower semicontinuous solutions of a large class of functional inequalities can be obtained from particular solutions of the related functional equations. Our main theorem reads as follows.

THEOREM.
Let $\lambda\colon I^2 \to (0,1)$ be a function and $n, m\colon I^2 \to I$ be continuous strict means. If there exists a non-constant and continuous solution $\phi\colon I \to \mathbb{R}$ of the equation

$$T^{\lambda}_{(n(x,y),m(x,y))}\phi = T^{\lambda}_{(x,y)}\phi, \qquad x, y \in I,$$

then ϕ is one-to-one, and a lower semicontinuous function $f\colon I \to \mathbb{R}$ satisfies the inequality

$$T^{\lambda}_{(n(x,y),m(x,y))}f \le T^{\lambda}_{(x,y)}f, \qquad x, y \in I,$$

if and only if $f \circ \phi^{-1}$ is convex on $\phi(I)$.

This result improves results presented in [1] and [2].

References

[1] J. Matkowski, M. Wróbel, *A generalized a-Wright convexity and related function equation*, Ann. Math. Silesianae **10** (1996), 7-12.

[2] Zs. Páles, *On two variable functional inequality*, C. R. Math. Rep. Acad. Sci. Canada, **10** (1988), 25-28.

Anna Bahyrycz
Pedagogical University, Kraków, Poland

A system of functional equations related to plurality functions

We consider the system of functional equations related to plurality functions:

$$f(x) \cdot f(y) \neq 0_m \quad \Longrightarrow \quad f(x+y) = f(x) \cdot f(y),$$

$$f(rx) = f(x),$$

where $f\colon \mathbb{R}(n) := [0, +\infty)^n \setminus \{0_n\} \to \mathbb{R}(m)$, $n, m \in \mathbb{N}$, $r \in \mathbb{R}(1)$ and

$$x + y := (x_1 + y_1, ..., x_k + y_k), \quad x \cdot y := (x_1 \cdot y_1, ..., x_k \cdot y_k), \quad rx := (rx_1, ..., rx_k)$$

for $x = (x_1, ..., x_k) \in \mathbb{R}(k)$, $y = (y_1, ..., y_k) \in \mathbb{R}(k)$.

We investigate systems of cones over $\mathbb{R}$, which are the parameter determining the solutions of this system.

Karol Baron
Silesian University, Katowice, Poland

Random–valued functions and iterative equations

As emphasized in [1; 0.3], iteration is the fundamental technique for solving functional equations of the form

$$F(x, \varphi(x), \varphi \circ f(x, \cdot)) = 0,$$

and iterates usually appear in the formulae for solutions. Moreover, many results may be interpreted in both ways: either as theorems about the behaviour of iterates, or as theorems about solutions of functional equations. In this survey we are interested in formulae of the form

$$\varphi(x) = \text{probability that the sequence } (f^n(x, \cdot))_{n \in \mathbb{N}} \text{ converges and its limit belongs to } B,$$

where the iterates f^n, $n \in \mathbb{N}$, are defined as in [1; 1.4] and B is a Borel set. Such formulae defining solutions of

$$\varphi(x) = \int_\Omega \varphi(f(x, \omega)) \mathrm{Prob}(d\omega)$$

are rather new in the theory of iterative functional equations, but as in more classical cases also results involving them may be read in two ways above described.

Reference

[1] Marek Kuczma, Bogdan Choczewski, Roman Ger, Iterative Functional Equations, Encyclopedia of Mathematics and its Applications, Vol. **32**, Cambridge University Press, Cambridge, 1990.

Nicole Brillouët-Belluot

Ecole Centrale de Nantes / Laboratoire de Mathématiques Jean Leray, Nantes, France

Some aspects of functional equations in physics (presented by **Joachim Domsta**)

Functional equations represent a way of modelling problems in physics. The physical problem is often directly stated in terms of one or several functional equations. However, a problem in physics may also be firstly described by a partial differential equation from which we derive a functional equation whose solutions solve the problem.

In this talk, I will present several examples of functional equations modelling physical problems in various fields of physics. In each example, I will mainly explain how the functional equation appears in the physical problem.

Janusz Brzdęk

Pedagogical University, Kraków, Poland

Fixed point results and stability of functional equations in single variable

Joint work with Roman Badora.

We show that stability of numerous functional equations in single variable is an immediate consequence of very simple fixed point results. We consider a generalization of the classical Hyers-Ulam stability (as suggested by T. Aoki, D.G. Bourgin and Th.M. Rassias), a modification of it, quotient stability (in the sense of R. Ger), and iterative stability.

Liviu Cădariu

"Politehnica" University of Timişoara, Romania

Fixed points method for the generalized stability of monomial functional equations

Joint work with Viorel Radu.

D.H. Hyers in 1941 gave an affirmative answer to a question of S.M. Ulam, concerning the stability of group homomorphisms in Banach spaces: *Let E_1 and E_2 be Banach spaces and $f\colon E_1 \to E_2$ be such a mapping that*

$$\|f(x+y) - f(x) - f(y)\| \le \delta \tag{1}$$

*for all $x, y \in E_1$ and a $\delta > 0$, that is f is $\delta-$*additive. *Then there exists a unique* additive $T\colon E_1 \to E_2$, *given by*

$$T(x) = \lim_{n\to\infty} \frac{f(2^n x)}{2^n}, \qquad x \in E_1, \tag{2}$$

which satisfies $\|f(x) - T(x)\| \le \delta$, $x \in E_1$.

T. Aoki, D. Bourgin and Th.M. Rassias studied the stability problem with unbounded Cauchy differences. Generally, the constant δ in (1) is replaced by a control function, $||\mathcal{D}_f(x,y)|| \le \delta(x,y)$, where, for example, $\mathcal{D}_f(x,y) = f(x+y) - f(x) - f(y)$ for Cauchy equation. The *stability estimations* are of the form $||f(x) - S(x)|| \leq \varepsilon(x)$, where *$S$ verifies the functional equation* $\mathcal{D}_S(x, y) = 0$, and for $\varepsilon(x)$ explicit formulae are given, which depend on the control δ as well as on the equation.

We use a fixed point method, initiated in [4] and developed, e.g., in [1], to give a generalized Ulam-Hyers stability result for functional equations in single variable and functions defined on groups, with values in sequentially complete locally convex spaces. This result is then used to obtain the generalized stability for some abstract monomial functional equations.

References

[1] L. Cădariu, V. Radu, *Fixed points and the stability of Jensen's functional equation*, J. Inequal. Pure and Appl. Math. **4(1)** (2003), Art.4 (http://jipam.vu.edu.au).

[2] D.H. Hyers, G. Isac, Th.M. Rassias, Stability of Functional Equations in Several Variables, Basel, 1998.

[3] V. Radu, *The fixed point alternative and the stability of functional equations*, Fixed Point Theory, Cluj-Napoca **IV(1)** (2003), 91-96.

Bogdan Choczewski

AGH University of Science and Techology, Kraków, Poland

Special solutions of an iterative functional inequality of second order

This a report on a joint work by Dobiesław Brydak, Marek Czerni and the speaker [1].

The inequality reads:

$$\psi[f^2(x)] \leq (p(x) + q(f(x))\psi[f(x)] - p(x)q(x)\psi(x), \tag{1}$$

where ψ is the unknown function. We aim at investigating these continuous solutions of (1) that behave at the fixed point of f like a prescribed "test" function T, in particular, like one from among the functions p, q or f.

Inequality (1) has been first studied by Maria Stopa [2].

References

[1] D. Brydak, B. Choczewski, M. Czerni, *Asymptotic properties of solutions of some iterative functional inequalities*, Opuscula Math., Volume dedicated to the memory of Professor Andrzej Lasota, in print.

[2] M. Stopa, *On the form of solutions of some iterative functional inequality*, Publ. Math. Debrecen **45** (1994), 371-377.

Jacek Chudziak

University of Rzeszów, Poland

On some property of the Gołąb-Schinzel equation

Let X be a linear space over a field K of real or complex numbers. Given nonempty subset A of X, we say that $a \in A$ is an algebraically interior point to A provided, for every $x \in X \setminus \{0\}$, there is an $r_x > 0$ such that

$$\{a + bx : |b| < r_x\} \subset A.$$

By $\mathrm{int}_a A$ we denote the set of all algebraically interior points to A.

We show that, rather surprisingly, in a class of functions $f\colon X \to K$ such that $F_f := \{x \in X : f(x) = 0\} \neq \emptyset$ and $\mathrm{int}_a(X \setminus F_f) \neq \emptyset$, the following two conditions are equivalent:

(i) $f(x + f(x)y) = 0$ if and only if $f(x)f(y) = 0$ for $x, y \in X$;

(ii) $f(x + f(x)y) = f(x)f(y)$ for $x, y \in X$.

Some consequences of this fact are also presented.

Marek Czerni
Pedagogical University, Kraków, Poland

Representation theorems for solutions of a system of linear inequalities

In the talk we present representation theorems for continuous solutions of a system of functional inequalities

$$\begin{cases} \psi[f(x)] \le g(x)\psi(x), \\ (-1)^p\psi[f^2(x)] \le (-1)^p g[f(x)]g(x)\psi(x) \end{cases} \tag{1}$$

where ψ is an unknown function, f, g are given functions, f^2 denotes the second iterate of f and $p \in \{0, 1\}$.

We assume the following hypotheses about the given functions f and g:

(H_1) The function $f\colon I \to I$ is continuous and strictly increasing in an interval $I = [0, a|$. ($a > 0$ may belong to I or not). Moreover $0 < f(x) < x$ for $x \in I^\star = I \setminus \{0\}$, $f(I) = I$.

(H_2) The function $g\colon I \to \mathbb{R}$ is continuous in I and $g(x) < 0$ for $x \in I$.

We shall be concerned with such solutions of (1) that for some fixed solution φ of a linear homogeneous functional equation

$$\varphi[f(x)] = g(x)\varphi(x)$$

or

$$\varphi[f(x)] = -g(x)\varphi(x)$$

the finite limit

$$\lim_{x \to 0^+} \frac{\psi(x)}{\varphi(x)}$$

exists.

Stefan Czerwik
Silesian University of Technology, Gliwice, Poland

Effective formulas for the Stirling numbers

It is known that Stirling numbers play important role in many areas of mathematics and applications. We shall present some results about the Stirling numbers. We introduce new definition of the Stirling numbers of second kind. Moreover, we shall present some effective formulas for the Stirling numbers of the first kind.

Zoltán Daróczy

University of Debrecen, Hungary

Nonconvexity and its application

Joint work with Zsolt Páles.

Let $I \subset \mathbb{R}$ be a nonempty open interval. The following characterization of a continuous nonconvex function $f : I \to \mathbb{R}$ is applicable for a number of questions in the theory of mean values.

THEOREM.
Let $f: I \to \mathbb{R}$ be a nonconvex continuous function on I. Then there exist $a \neq b$ in I such that

$$f(ta + (1-t)b) > tf(a) + (1-t)f(b)$$

holds for all $0 < t < 1$.

Judita Dascăl

University of Debrecen, Hungary

On a functional equation with a symmetric component

Let $I \subset \mathbb{R}$ be a nonvoid open interval and $r \neq 0, 1$, $q \in (0, 1)$, such that $r \neq q$, $r \neq \frac{1}{2}$ and $q \neq \frac{1}{2}$. In this presentation we give all the functions $f, g: I \to \mathbb{R}_+$ such that

$$f\left(\frac{x+y}{2}\right)[r(1-q)g(y) - (1-r)qg(x)] = \frac{r-q}{1-2q}[(1-q)f(x)g(y) - qf(y)g(x)]$$

for all $x, y \in I$. Our main result is the following.

If the functions $f, g: I \to \mathbb{R}_+$ are solutions of the above functional equation, then the following cases are possible:

(1) If $r \neq \frac{q^2}{q^2+(1-q)^2}$ and $r \neq \frac{q}{2q-1}$ then there exist constants $a, b \in \mathbb{R}_+$ such that

$$f(x) = a \quad \text{and} \quad g(x) = b \quad \text{for all } x \in I;$$

(2) If $r = \frac{q^2}{q^2+(1-q)^2}$ then there exists an additive function $A: \mathbb{R} \to \mathbb{R}$ and real numbers $c_1, c_2 > 0$ such that

$$g(x) = c_1 e^{A(x)} \quad \text{and} \quad f(x) = c_2 e^{2A(x)} \quad \text{for all } x \in I;$$

(3) If $r = \frac{q}{2q-1}$ then there exist real numbers d_1, d_2, d_3 such that

$$g(x) = \frac{1}{d_1 x + d_2} > 0 \quad \text{and} \quad f(x) = d_3 \frac{1}{d_1 x + d_2} > 0 \quad \text{for all } x, y \in I.$$

Conversely, the functions given in the above cases are solutions of the previous equation.

Joachim Domsta
Gdańsk University of Technology, Poland

An example of a group of commuting boosts

In this talk a construction of a particular group of invariant linear maps for $\mathbb{R}^{1+3}$ is given. The set of mappings is the same as the one of the Lorentz group, but the group action is not simply the composition of maps. It is chosen in such a way, that the group of rotations is the same, as in the Lorentz group. But all boosts form a subgroup, which does not hold in the Lorentz group. Additionally, this subgroup is abelian. An interesting fact is, that the one dimensional subgroups of the boosts are simultaneously (one dimensional) subgroups of the Lorentz group.

Piotr Drygaś
University of Rzeszów, Poland

Functional equations and effective conductivity in composite material with non perfect contact.

We consider a conjugation problem for harmonic functions in multiply connected circular domains. This problem is rewritten in the form of the $\mathbb{R}$-linear boundary value problem by using equivalent functional-differential equations in a class of analytic functions. It is proven that the operator corresponding to the functional-differential equations is compact in the Hardy-type space. Moreover, these equations can be solved by the method of successive approximations under some natural conditions. This problem has applications in mechanics of composites when the contact between different materials is imperfect. It is given information about effective conductivity tensor with fixed accuracy for macroscopic isotropy composite material.

Włodzimierz Fechner
Silesian University, Katowice, Poland

On some functional-differential inequalities related to the exponential mapping

We examine some functional-differential inequalities which are related to the exponential function. In particular, we show that its solutions can be written as a product of the exponential function and a convex mapping. Our results are closely connected with the Hyers-Ulam stability of functional-differential equations and, in particular, with some of the results obtained in 1998 by Claudi Alsina and Roman Ger in [1].

Reference

[1] C. Alsina, R. Ger, *On some inequalities and stability results related to the exponential function*, J. Inequal. Appl. **2** (1998), 373-380.

Roman Ger

Silesian University, Katowice, Poland

On functional equations related to functional analysis – selected topics

The talk focuses on the occurrence of various functional analysis aspects in the theory of functional equations and vice versa. Among others, the topics discussed concern representation theorems, generalizations of the Hahn-Banach type theorems and their geometric counterparts (separation results), characterizations of various kinds of Banach spaces, functional equations in Banach algebras, convex analysis, algebraic analysis, generalized polynomials, abstract orthogonalities and related equations, global isometries and their perturbations, stability and approximation theory and geometry of Banach spaces.

Dorota Głazowska

University of Zielona Góra, Poland

An invariance of the geometric mean in the class of Cauchy means

We determine all the Cauchy conditionally homogeneous mean-type mappings for which the geometric mean is invariant, assuming that one of the generators of Cauchy mean is a power function.

Grzegorz Guzik

AGH University of Science and Techology, Kraków, Poland

Derivations in some model of quantum gravity

A sketch of a role of derivations, i.e., linear operators satisfying a Leibnitz's rule in a new model of quantum gravity proposed by polish astrophysicist M. Heller and co-workers is presented. This promising model is an alternative to popular modern superstrings theories and it gives a hope to unification of relativity and quanta.

Konrad J. Heuvers

Michigan Technological University, USA

Some partial Cauchy difference equations for dimension two

Let G be an abelian group and X a vector space over the rationals. For $\Phi\colon G \to X$ its 1-st Cauchy difference is the function $K_2\Phi\colon G^2 \to X$ defined by

$$K_2\Phi(x_1, x_2) := \Phi(x_1 + x_2) - \Phi(x_1) - \Phi(x_2)$$

and in general, for $n = 2, 3, \ldots$, the $(n-1)$-th Cauchy difference of Φ is the function $K_n\Phi\colon G^n \to X$ defined by

$$K_n\Phi(x_1, \ldots, x_n) := \sum_{r=1}^{n} (-1)^{n-r} \sum_{|J|=r} \Phi(x_J)$$

where $\emptyset \neq J \subset I_n = \{1, \ldots, n\}$ and $x_J = \sum_{j \in J} x_j$. If $\Psi\colon G^n \to X$, then its i-th partial difference of order r ($r = 2, 3, \ldots$), $K_r^{(i)}\Psi\colon G^{n+r-1} \to X$, is its Cauchy difference of order r with respect to its i-th variable with all the others held fixed. For $n = 2$ and $i = 1, 2$ we have

$$K_2^{(1)}\Psi(x_1, x_2; x_3) = \Psi(x_1 + x_2, x_3) - \Psi(x_1, x_3) - \Psi(x_2, x_3)$$

and

$$K_2^{(2)}\Psi(x_1; x_2, x_3) = \Psi(x_1, x_2 + x_3) - \Psi(x_1, x_2) - \Psi(x_1, x_3).$$

In this talk the solutions of the following equations are given.

1. $K_2^{(1)} f_2 = K_2^{(2)} f_1$, where $f = \langle f_1, f_2 \rangle\colon G^2 \to X^2$ (a 2-dim "curl"$= 0$).
2. $K_2^{(1)} f_1 + K_2^{(2)} f_2 = 0$, where $f = \langle f_1, f_2 \rangle\colon G^2 \to X^2$ (a 2-dim "div"$= 0$).
3. $K_2^{(1)} f = K_2^{(2)} f$, where $f\colon G^2 \to X$.
4. $K_2^{(1)} f = \lambda K_2^{(2)} f$, where $\lambda \neq 0, 1$ and $f\colon G^2 \to X$. (Here the special case $\lambda = -i$ corresponds to a "Cauchy-Riemann equation".)

Eliza Jabłońska

University of Technology, Rzeszów, Poland

On Christensen measurability and a generalized Gołąb-Schinzel equation

Let X be a real linear space. We consider solutions $f\colon X \to \mathbb{R}$ and $M\colon \mathbb{R} \to \mathbb{R}$ of the functional equation

$$f(x + M(f(x))y) = f(x)f(y) \qquad \text{for } x, y \in X, \tag{1}$$

where f is bounded on a Christensen measurable nonzero set as well as f is Christensen measurable. Our results refer to some results of C.G. Popa and J. Brzdęk.

Justyna Jarczyk

University of Zielona Góra, Poland

On an equation involving weighted quasi-arithmetic means

We report on a progress made recently in studying solutions (φ, ψ) of the equation

$$\kappa x + (1-\kappa)y = \lambda\varphi^{-1}\left(\mu\varphi(x) + (1-\mu)\varphi(y)\right) + (1-\lambda)\psi^{-1}\left(\nu\psi(x) + (1-\nu)\psi(y)\right), \tag{1}$$

where $\kappa, \lambda \in \mathbb{R} \setminus \{0, 1\}$ and $\mu, \nu \in (0, 1)$. When $\kappa = \mu = \nu = 1/2$ all twice continuously differentiable solutions of (1) were found by D. Głazowska, W. Jarczyk, and J. Matkowski. Later Z. Daróczy and Zs. Páles determined all continuously differentiable solutions of (1) in the case $\kappa = \mu = \nu$.

Witold Jarczyk

University of Zielona Góra, Poland

Iterability in a class of mean-type mappings

Joint work with Janusz Matkowski.

Embeddability of a given pair of means in a continuous iteration semigroup of pairs of homogeneous symmetric strict means is considered.

Hans-Heinrich Kairies

Clausthal University of Technology, Clausthal-Zellerfeld, Germany

On Artin type characterizations of the Gamma function

E. Artin's monograph on the Gamma function contains two characterizations using the functional equation

$$f(x+1) = xf(x), \qquad x \in \mathbb{R}_+ \tag{F}$$

and the multiplication formula

$$f\left(\frac{x}{p}\right) f\left(\frac{x+1}{p}\right) \dots f\left(\frac{x+p-1}{p}\right) = (2\pi)^{\frac{1}{2}(p-1)} p^{\frac{1}{2}-x} f(x), \qquad x \in \mathbb{R}_+. \tag{M_p}$$

They read as follows.

Theorem A
Assume that $f\colon \mathbb{R}_+ \to \mathbb{R}_+$ *is continuously differentiable and satisfies* (F) *and* (M_p) *for some* $p \in \{2,3,4,\dots\}$. *Then* $f = \Gamma$.

Theorem B
Assume that $f\colon \mathbb{R}_+ \to \mathbb{R}_+$ *is continuous and satisfies* (F) *and* (M_p) *for every* $p \in \{2,3,4,\dots\}$. *Then* $f = \Gamma$.

We discuss both theorems with respect to their optimality.

Zygfryd Kominek

Silesian University, Katowice, Poland

Stability of a quadratic functional equation on semigroups

The stability problem of the functional equation of the form

$$f(x+2y) + f(x) = 2f(x+y) + 2f(y),$$

is investigated. We prove that if the norm of the difference between the left-hand side and the right-hand side of the equation is majorized by a function ω of two variables having some standard properties, then there exists a unique solution F of our equation and the norm of the difference between F and the given function f is controlled by a function depending on ω.

Krzysztof Król
Silesian University of Technology, Gliwice, Poland

Application of the least squares method and the decomposition method to solving functional equations

In the talk we consider the approximate solution of the linear functional equation

$$y[f(x)] = g(x)y(x) + F(x), \tag{1}$$

in the class of continuous functions.

We use the least squares method for finding the approximate solution of the equation (1). In this method the accurate solution of the equation (1) may be approximated by the function

$$y_n(x) = \sum_{j=1}^{n} p_j \Phi_j(x),$$

where $\Phi_j,\ j = 1, \ldots, n$, are given, continuous, linear independent functions, and coefficients p_j, $j = 1, \ldots, n$, are solutions of the system of equations

$$\sum_{j=1}^{n} p_j \int_a^b \Psi_i(x)\Psi_j(x)\,dx = \int_a^b \Psi_i(x)F(x)\,dx,$$

where $\Psi_i(x) = \Phi_i[f(x)] - g(x)\Phi_i(x)$ and $i = 1, \ldots, n$. We apply the least squares method to solving the exemplary equation.

Next we use the decomposition method for finding the approximate solution of the equation (1). At certain assumptions we show that the accurate solution of the equation (1) may be uniformly approximated by the function

$$y(x) = \sum_{n=0}^{\infty} \varphi_n(x),$$

where

$$\varphi_0(x) = -\frac{F(x)}{g(x)},\ \varphi_n(x) = \frac{\varphi_{n-1}(x)}{g(x)}, \quad n = 1, 2, \ldots$$

We prove that if there exists $0 \le \alpha < 1$ such that

$$\|\varphi_{n+1}\| \le \alpha\|\varphi_n\|, \quad n = 0, 1, \ldots$$

then the series $\sum_{n=0}^{\infty} \varphi_n(x)$ is uniformly convergent to the accurate solution of the equation (1). Finally, we apply the decomposition method to solving the exemplary equation.

Arkadiusz Lisak
University of Information Technology and Management in Rzeszów, Poland

Some remarks on solutions of functional equations stemming from trapezoidal rule

Joint work with Maciej Sablik.

The following functional equation (stemming from trapezoidal rule)

$$f_1(y) - g_1(x) = (y-x)[f_2(x) + f_3(sx+ty) + f_4(tx+sy) + f_5(y)]$$

with six unknown functions $g_1, f_i : \mathbb{R} \to \mathbb{R}$ for $i = 1, \ldots, 5$, where s and t are two fixed real parameters, has been solved by Prasanna K. Sahoo (University of Louisville, Louisville, USA). However, the solutions have been determined in particular for $s^2 \neq t^2$ (with $st \neq 0$) under high regularity assumptions on unknown functions (twice and four times differentiability). We solve this equation without any regularity assumptions on unknown functions for rational parameters s and t and with lesser regularity assumptions on unknown functions for real parameters s and t.

Fruzsina Mészáros
University of Debrecen, Hungary

Functional equations stemming from probability theory

Joint work with Károly Lajkó.

Special cases of the almost everywhere satisfied functional equation

$$g_1\left(\frac{x}{c(y)}\right)\frac{1}{c(y)}f_Y(y) = g_2\left(\frac{y}{d(x)}\right)\frac{1}{d(x)}f_X(x)$$

are investigated for the given positive functions c, d and unknown functions g_1, g_2, f_X and f_Y. This functional equation has important role in the characterization of distributions, whose conditionals belong to given scale families and have specified regressions.

Vladimir Mityushev
Pedagogical University, Kraków, Poland

Application of functional equations to composites and to porous media

Boundary value problems for multiply connected domains describe various physical phenomena in composites and porous media. One of the important constant of such problems constructed as a functional is the effective conductivity. Estimation of the effective conductivity can help to predict and to optimize properties of new created materials. It is shown that discussed boundary value problems can be effectively solved by reduction to iterative functional equations. New exact and approximate analytical formulae for the effective conductivity have been deduced. Further possible applications are discussed.

Takeshi Miura
Yamagata University, Yonezawa, Japan

A note on stability of Volterra type integral equation

Let $\mathbb{R}$ be the real number field and let X be a complex Banach space. Suppose that p is a continuous function from $\mathbb{R}$ to the complex number field. The purpose of this talk is to give a sufficient condition in order that the equation

$$f(t) - f(0) = \int_0^t p(s)f(s)\,ds \qquad (\forall t \in \mathbb{R}) \tag{$*$}$$

has the stability in the sense of Hyers-Ulam: for every $\varepsilon \geq 0$ and continuous map $f\colon \mathbb{R} \to X$ satisfying

$$\left\| f(t) - f(0) - \int_0^t p(s)f(s)\,ds \right\| \leq \varepsilon \qquad (\forall t \in \mathbb{R}),$$

there exists a solution $g\colon \mathbb{R} \to X$ of the equation $(*)$ such that

$$\|f(t) - g(t)\| \leq K\varepsilon \qquad (\forall t \in \mathbb{R}),$$

where K is a non-negative constant, depending only on the function p.

Janusz Morawiec
Silesian University, Katowice, Poland

On the set of probability distribution solutions of a linear equation of infinite order

Let $(\Omega, \mathcal{A}, P)$ be a probability space and let $\tau\colon \mathbb{R} \times \Omega \to \mathbb{R}$ be a function which is strictly increasing and continuous with respect to the first variable, measurable with respect to the second variable. We are interested in the following problem: How much we can say about the class of all probability distribution solutions of the equation

$$F(x) = \int_\Omega F(\tau(x,\omega))dP(\omega)?$$

Jacek Mrowiec
University of Bielsko-Biała, Poland

On nonsymmetric t-convex functions

Let $t \in (0,1)$ be a fixed number. It is known that if a function f defined on a convex domain D is t-convex, i.e., satisfies the condition

$$f(tx + (1-t)y) \leq tf(x) + (1-t)f(y), \qquad x, y \in D, \tag{$*$}$$

then it is a midconvex (Jensen-convex) function, i.e., it satisfies the inequality

$$f\left(\frac{x+y}{2}\right) \leq \frac{f(x)+f(y)}{2}$$

for all $x, y \in D$ (see [1] or [2], Lemma 1). Some years ago Zs. Páles has posed the following problem: Suppose that a function f satisfies the condition $(*)$ but only for $x < y$. Does this imply midconvexity of f? The partial answer to this question is given.

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Anna Mureńko
University of Rzeszów, Poland

A generalization of the Gołąb-Schinzel functional equation

We consider solutions $M, f : \mathbb{R} \to \mathbb{R}$ and $\circ : \mathbb{R}^2 \to \mathbb{R}$ of the functional equation

$$f(x + M(f(x))y) = f(x) \circ f(y),$$

under the following additional assumptions:

(a) f is continuous at a point;

(b) $M^{-1}(\{0\}) = \{0\}$;

(c) $\circ$ is commutative and associative.

Adam Najdecki
University of Rzeszów, Poland

On stability of some functional equation

Let $\mathcal{A}$ be a complex Banach algebra, S and T nonempty sets, and $h: T \to \mathcal{A}$. Moreover, let $a_j \in \mathbb{C}$ and $g_j: S \times T \to S$ for $j \in \mathbb{N}$. We are going to discuss the stability of the functional equation

$$\sum_{j=1}^{\infty} a_j f(g_j(s,t)) = h(t) f(s), \qquad s \in S, t \in T,$$

in the class of functions $f : S \to \mathcal{A}$.

Andrzej Olbryś
Uniwersytet Śląski, Katowice, Poland

On some functional inequality connected with t-Wright convexity and Jensen-convexity

Let $t \in (0,1)$ be a fixed number, $L(t)$-the smallest field containing the set $\{t\}$, and let X be a linear space over the field K, where $L(t) \subset K \subset R$. Let, moreover, $D \subset X$ be a $L(t)$-convex set, i.e., such set that $\alpha D + (1-\alpha)D \subset D$ for all $\alpha \in L(t) \cap (0,1)$.

In the talk we study connections between functions $f\colon D \to \mathbb{R}$ satisfying the inequality

$$\frac{f(tx+(1-t)y)+f((1-t)x+ty)}{2}+f(\frac{x+y}{2})\le f(x)+f(y), \qquad x,y\in D$$

and Jensen convex functions.

Boris Paneah

Technion, Haifa, Israel

On the solvability of the identifying problem for general functional operators with linear arguments

We start with a new problem for a general linear functional operator

$$(\mathcal{P}F)(x)=\sum_{j=1}^{N} c_j(x)F(a_j(x)), \qquad x\in D\subset\mathbb{R}^n,$$

where $F\in C(I,B),\ I=[-1,1],\ B$ a Banach space, a_j and c_j given functions. This problem is intimately connected in some sense with approximation theory and can be described shortly as follows: find a finite-dimensional subspace $\mathcal{K}\subset C(I,B)$, a one-dimensional manifold $\Gamma\subset D$ and a subspace $C_{\langle\tau\rangle}=C_{\langle\tau\rangle}(I,B)\subset C(I,B)$ such that for an arbitrary $\varepsilon>0$ the relation $|\mathcal{P}F|_{\langle\tau\rangle}<\varepsilon$ implies the inequality

$$\inf_{\varphi\in\mathcal{K}}|F-\varphi|_{\langle\tau\rangle}<c\varepsilon$$

with c a positive constant not depending on ε nor on F. If such a triple $(\mathcal{K},\ \Gamma,\ C_{\langle\tau\rangle})$ is found, we say that the identifying problem for the operator $\mathcal{P}$ is $(\Gamma,\ \mathcal{K})$ - solvable in the space $C_{\langle\tau\rangle}$. In particular, the well-known Hyers–Ulam result related to the functional Cauchy operator $\mathfrak{C}F=F(x,y)-F(x)-F(y)$ with $(x,y)\in\mathbb{R}^2$ can be reformulated as follows: the identifying problem for the operator $\mathfrak{C}$ is $(\mathbb{R}^2,\ \ker\mathfrak{C})$ - solvable in the space $C_{\langle\tau\rangle}$.

In the second part of the talk we give a solution of the identifying problem for a wide class of operators $\mathcal{P}$ with real c_j and linear functions a_j.

On the theory of the general linear functional operators with applications in analysis

In the talk we discuss the recent results related to the solvability and qualitative properties of solutions of the general linear functional equations

$$\sum_{j=1}^{N} c_j(x)F(a_j(x))=H(x),$$

where F are compact supported Banach-valued functions of a single variable and x are the points in a bounded domain $D\subset\mathbb{R}^n,\ n\ge 2$. When obtaining these results the new approach has been used. This is, first of all, the functional-analytic point of view which makes it possible to use the results and the methods of the classical functional analysis. Another novelty consists in systematical applying dynamical methods, based on the theory of the new dynamical systems introduced by the speaker (especially in connection with the problems in question). These results and methods will be considered at the first part of the talk. The second one is devoted to the (completely unexpected) connection of the above results with such divers fields of analysis as integral geometry, partial differential equations, and approximate solvability of the linear functional equations. The corresponding problems from these fields will be formulated (only some basic analysis is required for understanding) and their solutions will be given together with a list of unsolved problems (both in the theory of functional operators and in the applications).

Zsolt Páles
University of Debrecen, Hungary

Comparison theorems in various classes of generalized quasi-arithmetic means

Given a strictly increasing continuous function $f\colon I \to \mathbb{R}$, the A_f quasi-arithmetic mean of the numbers $x_1, \dots, x_n \in I$ is defined by

$$A_f(x_1, \dots, x_n) = f^{-1}\left(\frac{f(x_1) + \cdots + f(x_n)}{n}\right).$$

The following classical result has attracted the attention of many researchers during the last decades.

Theorem.
Let $f, g\colon I \to \mathbb{R}$ be continuous strictly increasing functions. Then the following conditions are equivalent:

- *For all $n \in \mathbb{N}$, $x_1, \dots, x_n \in I$,*
$$A_f(x_1, \dots, x_n) \leq A_g(x_1, \dots, x_n);$$
- *for all $p \in I$ there exists $\delta > 0$ such that, for all $x, y \in]p - \delta, p + \delta[$,*
$$A_f(x, y) \leq A_g(x, y);$$
- *$g \circ f^{-1}$ is convex;*
- *there exists a function $h : I \to \mathbb{R}$ such that, for all $x, y \in I$,*
$$f(x) - f(y) \leq h(y)(g(x) - g(y));$$
- *if f, g are twice differentiable with $f'g' \neq 0$ then, for all $x \in I$,*
$$\frac{f''(x)}{f'(x)} \leq \frac{g''(x)}{g'(x)}.$$

Our aim is to survey several extensions and of the above theorem related to various generalizations of quasi-arithmetic means.

Magdalena Piszczek
Pedagogical University, Kraków, Poland

On a multivalued second order differential problem with Jensen multifunctions

Let K be a closed convex cone with a nonempty interior in a real Banach space and let $cc(K)$ denote the family of all nonempty convex compact subsets of K. If $\{F_t : t \geq 0\}$ is a regular cosine family of continuous Jensen set-valued functions $F_t : K \to cc(K)$, $x \in F_t(x)$ for $t \geq 0$, $x \in K$ and $F_t \circ F_s = F_s \circ F_t$ for $s, t \geq 0$, then such family is twice differentiable and

$$DF_t(x)|_{t=0} = \{0\}, \qquad D^2F_t(x) = A_t(A(x) + D)$$

for $x \in K$ and $t \geq 0$, where $DF_t(x)$ denotes the Hukuhara derivative of $F_t(x)$ with respect to t, $\{A_t : t \geq 0\}$ is a regular cosine family of continuous additive multifunctions, $D \in cc(K)$ and $A(x) = D^2A_t(x)|_{t=0}$.

This result is a motivation for studying the existence and uniqueness of a solution

$$\Phi : [0, +\infty) \times K \to cc(K),$$

which is Jensen with respect to the second variable, of the following differentiable problem

$$\begin{aligned} \Phi(0, x) &= \Psi(x), \\ D\Phi(t, x)|_{t=0} &= \{0\}, \\ D^2\Phi(t, x) &= A_\Phi(t, H(x)), \end{aligned}$$

where $H, \Psi : K \to cc(K)$ are given continuous Jensen multifunctions, $D\Phi(t, x)$ denotes the Hukuhara derivative of $\Phi(t, x)$ with respect to t and A_Φ is the additive, with respect to the second variable, part of Φ.

Vladimir Protasov
Moscow State University, Russia

Self-similarity equations in L_p spaces

We consider functional difference equations with linear contractions of the argument (self-similarity equations). Let $L_p[0,1]$ be the space of vector-functions from the segment $[0,1]$ to $\mathbb{R}^d$ with the norm $\|v\|_p = (\int_0^1 |v(t)|^p \, dt)^{1/p}$. Suppose we have an arbitrary family of affine operators $\{\tilde{A}_1, \ldots, \tilde{A}_m\}$ in $\mathbb{R}^d$. We always assume this family to be irreducible (there is no common invariant affine subspace, different from the whole $\mathbb{R}^d$). Let us also have a partition of the segment $[0,1]$ with nodes $0 = b_0 < \ldots < b_m = 1$. We denote $\Delta_k = [b_{k-1}, b_k]$, $r_k = b_k - b_{k-1}$. The affine function $g_k(t) = t\, b_k + (1-t)\, b_{k-1}$ maps $[0,1]$ to the segment Δ_k. The *self-similarity operator* $\tilde{\mathbf{A}}$:

$$[\,\tilde{\mathbf{A}}\, v\,](t) \;=\; \tilde{A}_k\, v(g_k^{-1}(t)), \qquad t \in \Delta_k, \qquad k = 1, \ldots, m,$$

is defined on $L_1[0,1]$. The equation $\tilde{\mathbf{A}} v = v$ is called *self-similarity equation.* Special cases of such equations are applied in the ergodic theory, wavelets theory, approximation theory, probability, etc. Most of the classical fractal curves (such as Cantor singular function, Koch and de Rham curve, etc.) are solution of suitable self-similarity equations. Refinement equations from wavelets theory and approximation subdivision algorithms are also actually self-similarity equations.

We consider the following problem: what are the conditions on the operators $\{\tilde{A}_k\}$ and on the partition of the segment $[0,1]$ necessary and sufficient for the self-similarity equation to possess an L_p-solution? What can be said about the uniqueness and regularity of the solutions?

We derive a sharp criterion of solvability for these equations in the spaces L_p and C, compute the exponents of regularity and estimate the moduli of continuity. We show that the solution is always unique, whenever exists. The answers are given in terms of the so-called p-radius of the family of operators $\{\tilde{A}_k\}$. This, in particular, gives a geometric interpretation of the p-radius in terms of spectral radii of certain operators in the space $L_p[0,1]$.

Viorel Radu

Universitatea de Vest din Timişoara, Romania

The fixed point method to generalized stability of functional equations in normed and random normed spaces

D.H. Hyers in 1941 gave an affirmative answer to a question of S.M. Ulam, concerning the stability of group homomorphisms, for Banach spaces: *Let E_1 and E_2 be Banach spaces and $f\colon E_1 \to E_2$ be such a mapping that*

$$\|f(x+y) - f(x) - f(y)\| \leq \delta, \tag{1}$$

*for all $x, y \in E_1$ and a $\delta > 0$, that is f is $\delta-$*additive. *Then there exists a unique* additive $T\colon E_1 \to E_2$, *which satisfies $\|f(x) - T(x)\| \leq \delta$, $x \in E_1$. In fact,*

$$T(x) = \lim_{n\to\infty} \frac{f(2^n x)}{2^n}, \qquad x \in E_1, \tag{Hyers}$$

T. Aoki, D. Bourgin and Th.M. Rassias studied the stability problem with unbounded Cauchy differences: it is supposed that $||\mathcal{D}f(x,y)|| \leq \delta(x,y)$ and the *stability estimations* are of the form $||f(x) - S(x)|| \leq \varepsilon(x)$, where S is a solution, that is, it *verifies the functional equation* $\mathcal{D}S(x,y) = 0$, and for $\varepsilon(x)$ explicit formulae are given, which depend on the control δ as well as on the equation.

We discuss the generalized Ulam-Hyers stability for functional equations in abstract spaces and show how the stability results can be obtained by a fixed point method, initiated in (Radu [4], 2003) and developed in (Cădariu & Radu [2], 2004) as well as in subsequent papers.

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Ewa Rak
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Distributivity between uninorms and nullnorms

The problem of distributivity has been posed many years ago (cf. Aczel [1], pp. 318-319). A new direction of investigations is mainly concerned of distributivity between triangular norms and triangular conorms ([5] p.17). Recently, many authors have dealt with solution of distributivity equation for aggregation functions ([3]), fuzzy implications ([2], [10]), uninorms and nullnorms ([6], [7], [8], [9]), which are generalization of triangular norms and conorms.

Our consideration was motivated by intention of determining algebraic structures which have weaker assumptions than uninorms and nullnorms. In particular, the assumption of associativity is not necessary in consideration of distributivity equation. Moreover, if we omit commutativity assumption, consideration of the left and right distributivity conditions is reasonable. A characterization of such binary operations is interesting not only from a theoretical point of view, but also for their applications, since they have proved to be useful in several fields like fuzzy logic framework, expert system, neural networks or fuzzy quantifiers (cf. [4]).

Previous results about distributivity between uninorms and nullnorms can be obtained as simple corollaries.

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Themistocles M. Rassias
National Technical University of Athens, Greece

On some major trends in mathematics

In this talk I shall attempt to present some ideas regarding the present state and the near future of mathematics. Since assessments and any predictions in this field of science are necessarily subjective, I shall communicate to you the opinions of renowned contemporary mathematicians with some of whom I have recently come into contact. I will include of course the significant contribution of Polish mathematicians.

New and old problems in mathematical analysis

We present some new and old problems that are inspired by D. Hilbert problems [Göttinger Nachrichten (1900), 253–297, and the Bull. Amer. Math. Soc. **8** (1902), 437–479] and S. Smale problems [Mathematics: Frontiers and Perspectives, Mathematical Problems for the Next Century, International Mathematical Union, Amer. Math. Soc., 2000].

In particular emphasis is given to problems related to the representation of functions in several variables by means of functions of a smaller number of variables (J. d'Alembert, V. Arnold, N. Kolmogorov), A.D. Aleksandrov problem for isometric mappings and S.M. Ulam problem for approximate homomorphisms.

The interaction between analysis and geometry is discussed through old and new results, examples and further questions for future work.

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Maciej Sablik
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Generalized homogeneity of some means

We deal with means $M\colon I \times I \longrightarrow I$ which are $\circ$-homogenous, i.e., satisfying the equation

$$M(s \circ x, s \circ y) = s \circ M(x, y)$$

for all s, x, $y \in I$, where $\circ$ is a binary operation defined on $I \times I$. In particular, given a quasiarithmetic mean, we determine all continuous, associative and commutative operations $\circ$ with respect to which the mean is homogeneous. Also, we characterize given quasiarithmetic means as homogeneous with respect to a couple of suitable operations. This is a generalization of the well known result on characterization of the arithemtic mean as the only one which is homogeneous both with respect to ordinary multiplication and addition (see eg. J. Aczél, J. Dhombres, Functional Equations in Several Variables, Cambridge University Press, Cambridge, 1989).

The results have been partially obtained in collaboration with Małgorzata Pałys.

Ekaterina Shulman
Vologda State Pedagogical University, Russia

Some extensions of the Levi-Civita equation

Let T be a representation of a topological group G on a Banach space X. A vector x is called *finite* if there is a finite dimensional subspace $M \subset X$ such that $T_g x \in M$ for each $g \in G$. A finite dimensional subspace $L \subset X$ is called *special* if there is a finite dimensional subspace $M \subset X$ such that $T_g L \bigcap M \neq 0$, for each $g \in G$. We prove that a subspace is special if and only if it contains a finite vector. Using this result we describe continuous solutions $f_j(x)$ of the functional equation

$$\sum_{j=1}^{m} a_j(x) f_j(x+y) = \sum_{i=1}^{n} u_i(x) v_i(y)$$

which extends the well known Levi-Civita equation

$$f(x+y) = \sum_{i=1}^{n} u_i(x) v_i(y).$$

Justyna Sikorska
Silesian University, Katowice, Poland

On a conditional exponential functional equation and its stability

Joint work with Janusz Brzdęk.

We study a conditional functional equation of the form

$$\gamma(x+y) = \gamma(x-y) \quad \Longrightarrow \quad f(x+y) = f(x)f(y) \tag{$*$}$$

for a given function γ. Condition $(*)$ with $\gamma = \|\cdot\|$ is the so called isosceles orthogonally exponential functional equation. We show the form of the solutions and investigate the stability of the presented equation. Moreover, we study the pexiderized version of $(*)$.

Barbara Sobek

University of Rzeszów, Poland

Pexider equation on a restricted domain

Let $(X, +)$ be a uniquely 2-divisible Abelian topological group which has a base $\mathcal{B}$ of open neighbourhoods of 0 satisfying the following conditions:

(a) if $B \in \mathcal{B}$ and $x \in B$, then $\frac{x}{2} \in B$,

(b) if $B \in \mathcal{B}$ and $x \in X$, then there exists $n \in \mathbb{N} \cup \{0\}$ such that $\frac{x}{2^n} \in B$.

Assume that U is a nonempty, open and connected subset of $X \times X$. Let

$$U_1 := \{x : (x, y) \in U \text{ for some } y \in X\},$$

$$U_2 := \{y : (x, y) \in U \text{ for some } x \in X\}$$

and

$$U_+ := \{x + y : (x, y) \in U\}.$$

We consider the Pexider functional equation

$$f(x + y) = g(x) + h(y) \text{ for } (x, y) \in U,$$

where $f\colon U_+ \to K$, $g\colon U_1 \to K$ and $h\colon U_2 \to K$ are unknown functions and $(K, +)$ is an Abelian group. In particular, we improve Theorem 1 in [F. Radó, J.A. Baker, *Pexider's equation and aggregation of allocations*, Aequationes Math. **32** (1987), 227-239].

Paweł Solarz

Pedagogical University, Kraków, Poland

Some iterative roots for homeomorphisms with periodic points

Let $F\colon S^1 \to S^1$ be an orientation-preserving homeomorphism such that $\operatorname{Per} F$, the set of all periodic points of F, is nonempty. It is known that there is an integer $n > 1$ such that

$$\operatorname{Per} F = \{z \in S^1 : \ F^n(z) = z \text{ and } \forall_{0<k<n}\ F^k(z) \neq z\}.$$

If $\operatorname{Per} F \neq S^1$, the equation

$$G^m(z) = F(z), \qquad z \in S^1,$$

where $m \geq 2$, may not have continuous and orientation-preserving solutions. However, if $\gcd(m, n) = 1$, then there are infinitely many such solutions having periodic points of period n. These solutions depend on an arbitrary function. We give the general construction of these solutions.

Tomasz Szostok
Silesian University, Katowice, Poland

On an equation connected to Lobatto quadrature rule

Joint work with Barbara Koclęga-Kulpa.

Quadrature rules are used in numerical analysis for estimating integrals by the following formula

$$\int_x^y f(t)dt \approx (y-x)\sum_{i=1}^n \alpha_i f(a_i x + (1-a_i)y)$$

where the error term depends on the derivative of f. Further for the polynomials of certain degree (depending on the length and form of the quadrature considered) the above formula is exact. This means that polynomials satisfy equations of the type

$$F(y) - F(x) = (y-x)\sum_{i=1}^n \alpha_i f(a_i x + (1-a_i)y)$$

where F is the primitive function of f. In the current talk we solve an equation of this type with the right-hand side containing two endpoints and two other points from the interval $[x, y]$ which are symmetric with respect to the midpoint of this interval. Thus we deal with the equation

$$F(y) - F(x) = (y-x)[\alpha f(x) + \beta f(ax + (1-a)y) + \beta f((1-a)x + ay) + \alpha f(y)]$$

where functions $f, F\colon \mathbb{R} \to \mathbb{R}$ and constants $\alpha, \beta, a \in \mathbb{R}$ are unknown.

Jacek Tabor
Jagiellonian University, Kraków, Poland

Extensions of conditionally convex functions

Joint work with Józef Tabor.

Let $V \subset \mathbb{R}^N$ be a closed bounded convex set and let $f\colon \partial V \to \mathbb{R}$ be a continuous conditionally convex function, that is

$$f(\alpha x + (1-\alpha)y) \leq \alpha f(x) + (1-\alpha)f(y) \text{ for } \alpha \in [0,1], x, y \in V : [x, y] \subset V,$$

where $[x, y] = \{\alpha x + (1-\alpha)y : \alpha \in [0, 1]\}$. Then there exists a continuous convex function $F\colon V \to \mathbb{R}$ such that $F|_{\partial V} = f$.

We also show that the assumption that V is bounded is essential.

Józef Tabor
University of Rzeszów, Poland

Generalized approximate midconvexity

Joint work with Jacek Tabor.

Let X be a normed space and let $V \subset X$ be an open convex set. Let $\alpha: [0,\infty) \to \mathbb{R}$ be a given nondecreasing function. A function $f: V \to \mathbb{R}$ is $\alpha(\cdot)$-midconvex if

$$f\left(\frac{x+y}{2}\right) \le \frac{f(x)+f(y)}{2} + \alpha(\|x-y\|) \qquad \text{for all } x, y \in V.$$

We prove that if f is $\alpha(\cdot)$-midconvex and locally bounded at a point then

$$f(rx + (1-r)y) \le rf(x) + (1-r)f(y) + P_\alpha(r, \|x-y\|)$$

for $x, y \in V$, $r \in [0,1]$, where $P_\alpha: [0,1] \times [0,\infty) \to [0,\infty)$ is a function depending on α. Three different estimations of P_α are considered.

Aleksej Turnšek
University of Ljubljana, Slovenia

Mappings approximately preserving orthogonality

We present some results on orthogonality preserving and approximately orthogonality preserving mappings in the setting of inner product C^*-modules (Hilbert spaces). Some open questions are also considered.

Jian Wang
Fujian Normal University, P.R. China

The relation between isometric and affine operators on F^*-spaces

In this talk, we study the relation between isometries and affine operators on F^*-spaces, showing that for AL_β-spaces $(0 < \beta \le 1)$ E and F if E possesses a normalized complete disjoint atoms system $\{e_\gamma\}_{\gamma\in\Gamma}$, then an isometric embedding $T: E \to F$ with $T\emptyset = \emptyset$ is linear if and only if, for any $\gamma \in \Gamma$,

(i) $P_\gamma(TE) \subseteq \operatorname{span}(Te_\gamma)$ when $0 < \beta < 1$, and

(ii) $P_\gamma(TE) \subseteq \operatorname{span}(Te_\gamma)$ and $T(-e_\gamma) \in \operatorname{span}(Te_\gamma)$ when $\beta = 1$,

where P_γ is a principal band projection from F onto B_{Te_γ}. At the same time, we prove also that every onto isometry $T: (s)_p \to (s)_p$ (resp., $l^{(p_n)} \to l^{(p_n)}$, in particular, $l_\beta(\Gamma) \to l_\beta(\Gamma)$) is affine. For a number of results for isometric mappings one may see works of M. Day, Ding and Huang, and Th.M. Rassias.

Szymon Wąsowicz
University of Bielsko-Biała, Poland

On some inequalities between quadrature operators

In the class of 3–convex functions we establish the order structure of the set of six well known operators connected with an approximate integration: two–point and three–point Gauss–Legendre quadratures, Chebyshev quadrature, four–point and five–point Lobatto quadratures and the Simpson's Rule. We show that 12 (of 15 possible) inequalities are true while only 3 fail. For 5–convex functions the situation diametrally differs: only 3 inequalities hold and 12 fail. Among the considered inequalities at least one seems to be not trivial. To prove it we use some method connected with the spline approximation of convex functions of higher order.

Wirginia Wyrobek
Silesian University, Katowice, Poland

Measurable orthogonally additive functions modulo a discrete subgroup

Joint work with Tomasz Kochanek.

Under appropriate conditions on Abelian topological groups G and H, an orthogonality $\perp \subset G^2$ and a σ-algebra $\mathfrak{M}$ of subsets of G we decompose an $\mathfrak{M}$-measurable function $f\colon G \to H$ which is orthogonally additive modulo a discrete subgroup K of H into its continuous additive and continuous quadratic part (modulo K).